TRINITY COLLEGE

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 3&4

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	48	35
Section Two: Calculator-assumed	13	13	100	90	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.   You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.   It is recommended that you do not use pencil, except in diagrams.

6.   Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section One: Calculator-free **35% (48 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

(a) Determine $\int \cot(2x)\,dx$. (2 marks)

(b) Evaluate $\int_0^{\frac{\pi}{2}} \left(3 - \sec^2\left(\frac{x}{2}\right) + 3\tan^2\left(\frac{x}{2}\right)\right) dx$. (3 marks)
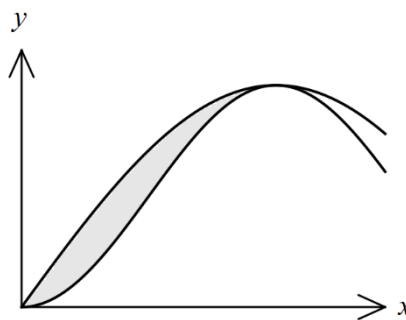
Question 2 (5 marks)

The curves $y = 3\sin\left(\frac{x}{2}\right)$ and $y = 3\sin^2\left(\frac{x}{2}\right)$ are shown to the right.

Determine the area of the shaded region trapped between the curves.

Question 3 **(6 marks)**

Consider the system of equations (where c is a constant) given by

$$x + y + cz + 1 = 0$$
$$2x + y - z - 5 = 0$$
$$2x + 2y + z = 0$$

(a) When $c = 1$, solve the system of equations and interpret your solution geometrically.

(4 marks)

(b) State the value of c for which the system of equations has no solution and explain the geometric interpretation of this. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 4   **(6 marks)**

Consider $f(z) = z^4 + kz^3 + 9z^2 - 8z + 20$, where k is a real constant.

The equation $f(z) = 0$ has a solution $z = 1 - 2i$.

(a)   State a second solution to $f(z) = 0$.   (1 mark)

(b)   Deduce that $k = -2$.   (3 marks)

(c)   Determine all other solutions of the equation $f(z) = 0$.   (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 5   **(7 marks)**

Functions f, g and h are defined by $f(x) = \sqrt{25 - x}$, $g(x) = \dfrac{10}{x}$ and $h(x) = f \circ g(x)$.

(a)   Determine the defining rule for $h(x)$ and state its domain.   (4 marks)

(b)   Determine the defining rule for $h^{-1}(x)$ and state its range.   (3 marks)

Question 6 (6 marks)

(a) Express $\dfrac{6}{(u-3)(u+3)}$ in the form $\dfrac{a}{u-3} + \dfrac{b}{u+3}$. (2 marks)

(b) Use the substitution $u^2 = x + 7$ to determine the indefinite integral I shown below in the form $\ln\big(g(x)\big) + c$ for $x > 2$. (4 marks)

$$I = \int \frac{3}{(x-2)\sqrt{x+7}}\,dx$$

Question 7   **(6 marks)**

The functions $f(x)$ and $g(x)$ are polynomials in x of degree two and degree one respectively, and their quotient is the rational function $h(x) = \dfrac{f(x)}{g(x)}$.

The graph of $y = h(x)$ passes through the point $(0, -1)$, has vertical asymptote $x = -3$ and has roots at $x = -2$ and $x = 3$.

Determine the equation of the other asymptote of the graph of $y = h(x)$.

Question 8 (7 marks)

(a) Solve the equation $z^3 = 8\operatorname{cis}\left(\dfrac{2\pi}{3}\right)$, where $-\pi < \arg(z) < 0$. (2 marks)

(b) $1, w$ and w^2 are the three cube roots of unity.

(i) State the value of $1 + w + w^2$ and the value of w^3. (1 mark)

Let $u = (1 + 3w + w^2)^2$ and $v = (1 + w + 3w^2)^2$.

(ii) Show that $u + v = -4$ and $uv = 16$. (4 marks)

End of questions

Supplementary page

Question number: _____